1st-Year Mathematics: Complex Analysis

Problem Sheet 1 2017

For tutorials

1. Consider the polar form of a complex number $z = re^{i\theta}$. Show that $(z^2)^* = (z^*)^2$.

2. (a) Consider the polynomial $az^2 + bz + c$, where a, b, and c may be complex and $a \neq 0$. Given that z_0 and z_0^* are distinct solutions to $az^2 + bz + c = 0$ and thus are also solutions to $z^2 + (b/a)z + (c/a) = z^2 + b'z + c' = 0$, show that b' and c' must be real. Conversely, if a, b and c are all real, deduce that z^* is a solution of $az^2 + bz + c = 0$.

   (b) Let $b = c = 1$ and $a = t$, $t > \frac{1}{4}$. Make a graph of the pattern that the solutions to this equation traces out in the complex plane (hint: work out $|z+1|$).

   (c) Derive functions $a(t)$, $b(t)$ and $c(t)$ for which the solutions to the equation traces out a pattern which is rotated by 90° counterclockwise around the origin with respect to the previous pattern (n.b. it is no longer the case that a, b and c are all real. Why?). How can you achieve an arbitrary rotation?

Homework

1. Find the real and imaginary parts of:

   (a) $8+3i$ (b) $4-15i$ (c) $\cos\theta - i\sin\theta$

   (d) i^2 (e) $i(2-5i)$ (f) $(1+2i)(2-3i)$

2. Write each of the following expressions as a complex number in the form $x+iy$:

   (a) $(5-i)(2+3i)$ (b) $(3-4i)(3+4i)$ (c) $(1+2i)^2$

   (d) $\dfrac{10}{4-2i}$ (e) $\dfrac{3-i}{4+3i}$ (f) $\dfrac{1}{i}$

3. Define $z = (5+7i)(5+bi)$.

   (a) If b and z are both real, find b.

   (b) If $\mathrm{Im}(b) = \frac{4}{5}$, and z is pure imaginary, find $\mathrm{Re}(b)$.

4. Plot the following complex numbers in the complex plane:

   (a) $2i$ (b) $-3+i2$ (c) $(-3+i2)^*$ (d) $\dfrac{1+i}{\sqrt{2}}$

5. Write the following numbers in polar form:

   (a) i (b) $-i$ (c) $1+i$ (d) $1-i\sqrt{3}$

6. Write the following complex numbers in Cartesian coordinates:

   (a) $e^{-3\pi i/4}$ (b) $e^{5\pi i/4}$ (c) $3e^{i}$ (d) $\dfrac{1}{\sqrt{3}e^{\pi i/3}}$