Complex Analysis (Prof. M. McCall)
Multiple Choice Sheet 3

1. An oscillator whose displacement is $x(t)$, and whose natural frequency is ω_0, is critically damped. At $t = 0$ the displacement is x_0 and it is at rest. For times $t \ll 2\pi/\omega_0$ the displacement follows approximately

   (a)
   $$x(t) \approx x_0\left[1 + (\omega_0 t)^2\right] .$$

   (b)
   $$x(t) \approx x_0\left[1 - (\omega_0 t)^2/2\right] .$$

   (c)
   $$x(t) \approx x_0\left[1 - (\omega_0 t)^2\right] .$$

   (d)
   $$x(t) \approx x_0\left[1 - (\omega_0 t)\right]^2 .$$

   [2 marks]

2. For times $t \gg 2\pi/\omega_0$ the displacement of the above system follows

   (a)
   $$x(t) = x_0 \cos(\omega_0 t) .$$

   (b)
   $$x(t) = x_0 \omega_0 t e^{\omega_0 t} .$$

   (c)
   $$x(t) = x_0 \sin(\omega_0 t) .$$

   (d)
   $$x(t) = x_0 \omega_0 t e^{-\omega_0 t} .$$

   [2 marks]

3. The displacement of an oscillator follows the equation $d^2x/dt^2 + \gamma dx/dt + \omega_0^2 x = 0$, with $\gamma = 1\ \mathrm{s}^{-1}$ and $\omega_0 = \sqrt{10}\ \mathrm{rads}^{-1}$. The amplitude of the oscillations will have diminished by a factor of 10 after a time

   (a)
   $$t = 7.0\ \mathrm{s}$$

   (b)
   $$t = 5.3\ \mathrm{s}$$

   (c)
   $$t = 4.6\ \mathrm{s}$$

   (d)
   $$t = 10.2\ \mathrm{s}$$

[2 marks]

4. During the time calculated in the previous question the oscillator will have completed approximately n cycles. The best estimate for n is

   (a)
   $$n = 2\,.$$

   (b)
   $$n = 7\,.$$

   (c)
   $$n = 1\,.$$

   (d)
   $$n = 15\,.$$

   [2 marks]

5. If A and B are complex constants, the general solution to the equation

   $$\frac{d^2y}{dx^2} + (1-2i)\frac{dy}{dx} - (i-1)y = 0$$

   is

   (a)
   $$y(x) = \left(Ae^{\sqrt{7}x/2} + Be^{-\sqrt{7}x/2}\right) e^{\left(-\frac{1}{2}+i\right)x}\,.$$

   (b)
   $$y(x) = \left(Ae^{i\sqrt{7}x/2} + Be^{-i\sqrt{7}x/2}\right) e^{\left(\frac{1}{2}-i\right)x}\,.$$

   (c)
   $$y(x) = \left(Ae^{\sqrt{7}x/2} + Be^{-\sqrt{7}x/2}\right) e^{-x/2}\,.$$

   (d)
   $$y(x) = \left(Ae^{i\sqrt{7}x/2} + Be^{-i\sqrt{7}x/2}\right) e^{\left(-\frac{1}{2}+i\right)x}\,.$$

   [2 marks]