1st-Year Mathematics: Complex Analysis

Multiple choice 1 - Solutions 2017

1. The complex conjugate z^* of a complex number $z = x + iy$ is $z^* = x - iy$. Thus,
   (a) $(5+3i)^* = 5-3i$
   (b) $\left[-(7-2i)\right]^* = (-7+2i)^* = -7-2i$
   (c) $(i^2)^* = (-1)^* = -1$
   (d) $\left[(2-3i)(i+7)\right]^* = (2i+14+3-21i)^* = (17-19i)^* = 17+19i$

2. (a) This calculation is best carried out in the polar representation:

$$z_1 = 2\sqrt{2}e^{i\theta}, \qquad \cos\theta = \frac{\sqrt{2}}{2}, \qquad \sin\theta = \frac{\sqrt{2}}{2},$$

so $\theta = \frac{1}{4}\pi$. Hence,

$$z_1^{10} = (2\sqrt{2})^{10} e^{10i\theta} = 8^5 e^{\frac{10}{4}\pi i} = 8^5 e^{\frac{1}{2}i\pi} = 8^5 i = 0 + 32768\, i.$$

(b) This calculation is best carried out in the Cartesian representation. We first determine z_2^4:

$$z_2^4 = (z_2^2)^2 = \left[(-1+3i)^2\right]^2 = (1-6i-9)^2$$

$$= (-8-6i)^2 = 64 + 96i - 36 = 28 + 96i.$$

We then have to take the inverse

$$z_2^{-4} = \frac{1}{z_2^4} = \frac{28-96i}{(28+96i)(28-96i)} = \frac{1}{10000}(28-96i).$$

(c) This will just be the complex conjugate of part (a). So

$$(z_1^*)^{10} = (z_1^{10})^* = 0 - 32768\, i.$$

3. The polynomial must be of the form

$$(z-i)(z-e^{i\pi/4})(z-e^{i3\pi/4}).$$

For multiplying it out, it is easiest to keep the polar form

$$z^3 + (-e^{i3\pi/4} - i - e^{i\pi/4})z^2 + (-1 + ie^{i3\pi/4} + ie^{i\pi/4})z + i$$

and then convert to Cartesian coordinates giving

$$z^3 - i(1+\sqrt{2})z^2 - (1+\sqrt{2})z + i. \quad \text{i.e. answer (b).}$$