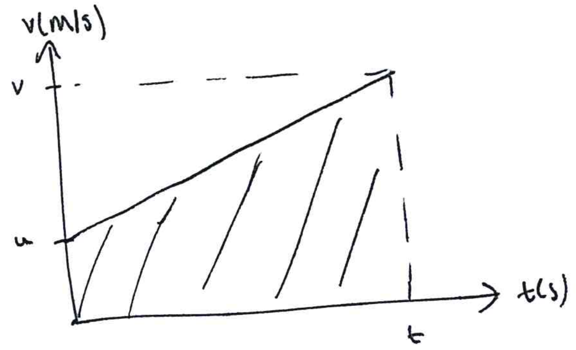
Deriving SUVAT without calculus



From this graph we can find the gradient of the line and the area under the line.

Gradient = acceleration = $\frac{v-u}{t}$ ⇒ $at = v - u$ so $v = u + at$ ①

Area = displacement = $\frac{1}{2}(u+v)t$ ⇒ $s = \left(\frac{u+v}{2}\right)t$ ②

From eq ① we see that

$v = u + at$, ⓐ

$u = v - at$ ⓑ

and $t = \frac{v-u}{a}$. ⓒ

(by simple rearrangement)

So let us sub into eq ②:

$$s = \left[\frac{u + (u+at)}{2}\right]t = \frac{(2u+at)t}{2} = \frac{2ut + at^2}{2} = ut + \frac{1}{2}at^2$$

So $s = ut + \frac{1}{2}at^2$ ③

$$s = \left[\frac{(v-at)+v}{2}\right]t = \frac{2vt - at^2}{2} = \frac{2vt}{2} - \frac{at^2}{2} = vt - \frac{1}{2}at^2$$

So $s = vt - \frac{1}{2}at^2$ ④

$$s = \left[\frac{u+v}{2}\right]\left[\frac{v-u}{a}\right] \Rightarrow 2as = (u+v)(u-v) \Rightarrow v^2 - u^2 = 2as$$

$v^2 = u^2 + 2as$ ⑤

Deriving SUVAT with calculus

We know that $\underline{r} = \int \underline{v}\, dt$ ① and $\underline{v} = \int \underline{a}\, dt$ ②

from ② : $\underline{v} = \underline{a}t + c$ (by integrating)

let us say that when $t = 0$, $\underline{v} = \underline{u}$

so $\underline{u} = c$

Hence $\underline{v} = \underline{u} + \underline{a}t$ or $v = u + at$ in scalar form.

from ① : $\underline{r} = \int \underline{v}\, dt$; we know now that $\underline{v} = \underline{u} + \underline{a}t$, so

$\underline{r} = \int (\underline{u} + \underline{a}t)\, dt$

$\underline{r} = \underline{u}t + \frac{1}{2}\underline{a}t^2 + c$ (by integrating)

let us say that at $t=0$, $\underline{r} = 0$

so $0 = c$

Hence $\underline{r} = \underline{u}t + \frac{1}{2}\underline{a}t^2$ or $s = ut + \frac{1}{2}at^2$ in scalar form.