Complex Analysis (Prof. M. McCall)
Assessed Problem Sheet

1. Write each of the following expressions in the form $a + ib$, where a and b are real numbers:

   (a) $(3 - i)(2 + i)$. [1 mark]

   (b) $i(2 - i)^*$. [1 mark]

   (c) $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^3$. [1 mark]

   (d) Principal value of $\operatorname{arc}\tanh(-i)$. [1 mark]

2. Show that the solution to $e^{iz} = 3i$ is $z = \left(\frac{\pi}{2} + 2\pi k\right) - i\ln 3$, where $k \in \mathbb{Z}$. [1 mark]

3. Show that there is no complex number z such that $\tan z = i$. [1 mark]

4. (a) Show that for real constants a and b,

$$\int e^{-ax}\cos bx\, dx = e^{-ax}\frac{-a\cos bx + b\sin bx}{a^2 + b^2},$$

   [1 mark]

   and

   (b)

$$\int e^{-ax}\sin bx\, dx = -e^{-ax}\frac{a\sin bx + b\cos bx}{a^2 + b^2}.$$

   [1 mark]

   (Hint: evaluate $\int e^{-(a+ib)x}dx$.)

5. (a) Taking, as necessary, $\ln i = i\pi/2$, show that the sequence $i, i^i, (i^i)^i, \cdots$ repeats. [1 mark]

   (b) Plot all the points of the sequence in part (a) on the Argand diagram labelling the coordinates of each point. [1 mark]