Pearson Edexcel Level 3

GCE Mathematics

Advanced

Paper 2: Pure Mathematics

Mock paper Spring 2018 Time: 2 hours	Paper Reference(s) 9MA0/02

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 14 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Answer ALL questions.

1.

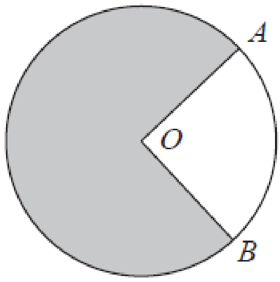

Figure 1

Figure 1 shows a circle with centre O. The points A and B lie on the circumference of the circle.

The area of the major sector, shown shaded in Figure 1, is 135 cm^2. The reflex angle AOB is 4.8 radians.

Find the exact length, in cm, of the minor arc AB, giving your answer in the form $a\pi + b$, where a and b are integers to be found.

(Total for Question 1 is 4 marks)

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2. (a) Given that θ is small, use the small angle approximation for $\cos\theta$ to show that

$$1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2.$$

(3)

Adele uses $\theta = 5°$ to test the approximation in part (a).

Adele's working is shown below.

Using my calculator, $1 + 4\cos(5°) + 3\cos^2(5°) = 7.962$, to 3 decimal places.
Using the approximation $8 - 5\theta^2$ gives $8 - 5(5)^2 = -117$
Therefore, $1 + 4\cos\theta + 3\cos^2\theta \approx 8 - 5\theta^2$ is not true for $\theta = 5°$.

(b) (i) Identify the mistake made by Adele in her working.

(ii) Show that $8 - 5\theta^2$ can be used to give a good approximation to $1 + 4\cos\theta + 3\cos^2\theta$ for an angle of size 5°.

(2)

(Total for Question 2 is 5 marks)

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3. A cup of hot tea was placed on a table. At time t minutes after the cup was placed on the table, the temperature of the tea in the cup, θ °C, is modelled by the equation

$$\theta = 25 + Ae^{-0.03t}$$

where A is a constant.

The temperature of the tea was 75 °C when the cup was placed on the table.

(*a*) Find a complete equation for the model.

(1)

(*b*) Use the model to find the time taken for the tea to cool from 75 °C to 60 °C, giving your answer in minutes to one decimal place.

(2)

Two hours after the cup was placed on the table, the temperature of the tea was measured as 20.3 °C.

Using this information,

(*c*) evaluate the model, explaining your reasoning.

(1)

(Total for Question 3 is 4 marks)

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4. (*a*) Sketch the graph with equation

$$y = |2x - 5|,$$

stating the coordinates of any points where the graph cuts or meets the coordinate axes.

(2)

(*b*) Find the values of x which satisfy

$$|2x - 5| > 7.$$

(2)

(*c*) Find the values of x which satisfy

$$|2x - 5| > x - \frac{5}{2}.$$

Write your answer in set notation.

(2)

(Total for Question 4 is 6 marks)

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5. The line l has equation $3x - 2y = k$, where k is a real constant.

Given that the line l intersects the curve with equation $y = 2x^2 - 5$ at two distinct points, find the range of possible values for k.

(Total for Question 5 is 5 marks)

6.

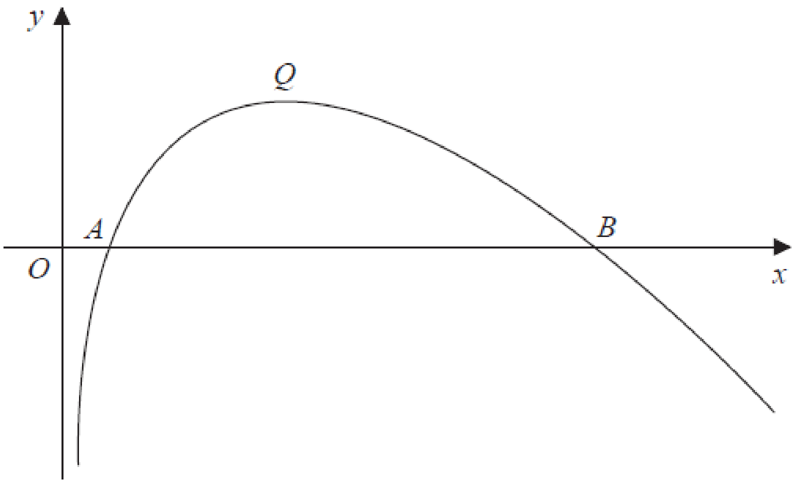

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (8 - x) \ln x$, $x > 0$.

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 2.

(a) Find the x coordinate of A and the x coordinate of B.

(1)

(b) Show that the x-coordinate of Q satisfies $x = \dfrac{8}{1 + \ln x}$.

(4)

(c) Show that the x-coordinate of Q lies between 3.5 and 3.6

(2)

(d) Use the iterative formula $x_{n+1} = \dfrac{8}{1 + \ln x_n}$ $n \in$ with $x_1 = 3.5$ to find

(i) the value of x_5 to 4 decimal places,

(ii) the x-coordinate of Q accurate to 2 decimal places.

(2)

(Total for Question 6 is 9 marks)

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7. A bacterial culture has area p mm^2 at time t hours after the culture was placed onto a circular dish.

A scientist states that at time t hours, the rate of increase of the area of the culture can be modelled as being proportional to the area of the culture.

(*a*) Show that the scientist's model for p leads to the equation $p = ae^{kt}$, where a and k are constants.

(4)

The scientist measures the values for p at regular intervals during the first 24 hours after the culture was placed onto the dish. She plots a graph of $\ln p$ against t and finds that the points on the graph lie close to a straight line with gradient 0.14 and vertical intercept 3.95.

(*b*) Estimate, to 2 significant figures, the value of a and the value of k.

(3)

(*c*) Hence show that the model for p can be rewritten as $p = ab^t$, stating, to 3 significant figures, the value of the constant b.

(2)

With reference to this model,

(*d*) (i) interpret the value of the constant a,

(ii) interpret the value of the constant b.

(2)

(*e*) State a long term limitation of the model for p.

(1)

(Total for Question 7 is 12 marks)

8.

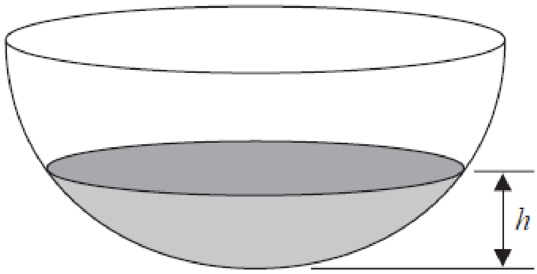

Figure 3

A bowl is modelled as a hemispherical shell as shown in Figure 3.

Initially the bowl is empty and water begins to flow into the bowl.

When the depth of the water is h cm, the volume of water, V cm^3, according to the model is given by

$$V = \frac{1}{3}\pi h^2(75 - h), \qquad 0 \leq h \leq 24.$$

The flow of water into the bowl is at a constant rate of 160π cm^3 s^{-1} for $0 \leq h \leq 12$.

(*a*) Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 10$.

(5)

Given that the flow of water into the bowl is increased to a constant rate of 300π cm^3 s^{-1} for $12 < h \leq 24$,

(*b*) find the rate of change of the depth of the water, in cm s^{-1}, when $h = 20$

(2)

(Total for Question 8 is 7 marks)

9. A circle with centre A (3, –1) passes through the point P (–9, 8) and the point Q (15, –10).

(*a*) Show that PQ is a diameter of the circle.

(2)

(*b*) Find an equation for the circle.

(3)

A point R also lies on the circle.

Given that the length of the chord PR is 20 units,

(*c*) find the length of the shortest distance from A to the chord PR, giving your answer as a surd in its simplest form.

(2)

(*d*) Find the size of angle ARQ, giving your answer to the nearest 0.1 of a degree.

(2)

(Total for Question 9 is 9 marks)

10.

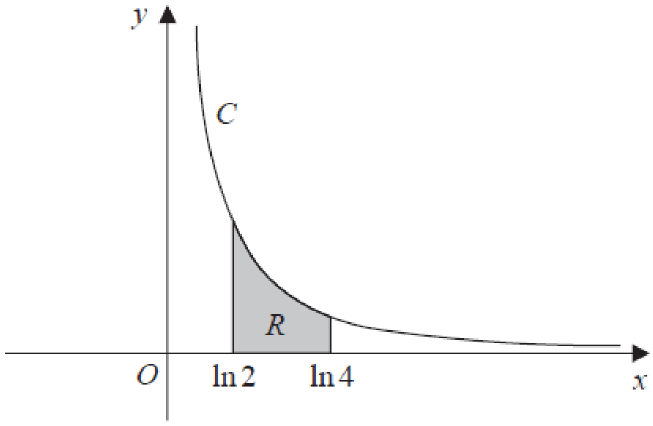

Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \ln(t+2), \quad y = \frac{1}{t+1}, \qquad t > -\frac{2}{3}.$$

(*a*) State the domain of values of x for the curve C.

(1)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = \ln 2$, the x-axis and the line with equation $x = \ln 4$

(*b*) Use calculus to show that the area of R is $\ln \frac{3}{2}$.

(8)

(Total for Question 10 is 9 marks)

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11. The second, third and fourth terms of an arithmetic sequence are $2k$, $5k - 10$ and $7k - 14$ respectively, where k is a constant.

Show that the sum of the first n terms of the sequence is a square number.

(Total for Question 11 is 5 marks)

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12. A curve C is given by the equation

$$\sin x + \cos y = 0.5, \qquad \frac{\pi}{2} \leq x < \frac{3\pi}{2}, \quad -\pi < y < \pi.$$

A point P lies on C. The tangent to C at the point P is parallel to the x-axis.

Find the exact coordinates of all possible points P, justifying your answer.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(Total for Question 12 is 7 marks)

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13. (*a*) Show that

$$\operatorname{cosec} 2x + \cot 2x \equiv \cot x, \quad x \neq 90n°, \quad n \in \mathbb{Z}.$$

(5)

(*b*) Hence, or otherwise, solve, for $0 \leq \theta < 180°$,

$$\operatorname{cosec}(4\theta + 10°) + \cot(4\theta + 10°) = \sqrt{3}.$$

You must show your working.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(5)

(Total for Question 13 is 10 marks)

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14. Kayden claims that $3^x \geq 2^x$.

(i) Determine whether Kayden's claim is always true, sometimes true or never true, justifying your answer.

(2)

(ii) Prove that $\sqrt{3}$ is an irrational number.

(6)

(Total for Question 14 is 8 marks)

TOTAL FOR PAPER IS 100 MARKS

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